Radiant School Mahendranagar

Project Work (Activities) for Grade XI 2078

1. INTRODUCTION

Mathematics is the building block for everything in our daily lives. For the effective teaching and learning , "Learning by doing" is of great importance as the experiences gained remains permanently affixed in the mind of the child. So, mathematics should be visualized as a vehicle to train a child to think and analyze logically. This is possible only if there is an opportunity of creating a scope of exploring, verifying and experimenting upon mathematical results by students themselves. Thus, there is a need of adopting activities.

2. OBJECTIVES

The general objectives of the study are:

- To understand the mathematical concepts and formulae through activities.
- To conduct scientific experiments for verification and exploration.
- To provide the students the scope for interaction, communication and representations of mathematical ideas by practicing processes.
- To develop mathematical skills that will prepare the students for the challenges of the future.
- To learn certain concepts using concrete objects and verify many mathematical fact and properties.
- To provide opportunity to the students to do certain calculations using tables, calculator, models, charts etc.
- To explain visually some abstract concepts, ideas etc.

3. MAIN TEXT

The main text should be organized under the following headings:

1. Title (in block letters)
2. Objectives
3. Material Required
4. Theory
5. Procedure
6. Observation
7. Conclusion/ Result
8. Applications
9. References

4. SOME EXAMPLES

The following are some illustrated examples of project works in mathematics of Grade XI :

EXAMPLE 1 (Activity No. 1)

1. **Title:**

   TO VERIFY AND UNDERSTAND THE WELL-KNOWN FORMULA $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$.

2. **Objectives:**

   (i) To understand the mathematical concept of the formula $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$.

   (ii) To learn the concept of limits by verifying the formula $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$.

3. **Material Required:**

   White paper, pencil, ruler, sharpener etc.

4. **Theory:**

   When x approaches 0 from either side but $x \neq 0$, the function $\frac{\sin x}{x}$ approaches 1, where x is measured in radian.

   Mathematically,

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

5. **Procedure:**

   (i) Put the calculator in radian mode by performing the operation

$$\boxed{\text{shift}} \longrightarrow \boxed{\text{mod}} \longrightarrow \boxed{4}$$

   (ii) Fix the calculator for 7 digits after decimal by adopting the operation

$$\boxed{\text{shift}} \longrightarrow \boxed{\text{mod}} \longrightarrow \boxed{6} \longrightarrow \boxed{7}$$

   (iii) Starting with $x = 0.1$, find the corresponding value of $\frac{\sin x}{x}$ using calculator and prepare the table as shown below:

x	0.1	0.05	0.01	0.005	0.001	$\cdots$
$\frac{\sin x}{x}$	0.9983342	0.9995834	0.9999833	0.9999958	0.9999998	$\cdots$

Table 1

   (iv) Starting with $x = -0.5$, find the corresponding value of $\frac{\sin x}{x}$ using calculator and prepare the table as given below:

x	- 0.5	- 0.1	- 0.01	- 0.001	$\cdots$
$\frac{\sin x}{x}$	0.9588511	0.9983342	0.9999833	0.9999998	$\cdots$

Table 2

6. **Observation:**

   In Table 1, as x approaches 0, $\frac{\sin x}{x}$ approaches 1.

In Table 2, as x approaches 0 , $\frac{\sin x}{x}$ approaches 1.

7. Conclusion:

As , x approaches 0 from either side , $\frac{\sin x}{x}$ approaches 1. So

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

8. Applications:

This activity can be used in evaluating limits involving trigonometric functions.

9. References:

(i) Bajracharya, B.C.; "Basic Mathematics" Grade XI; Sukunda Pustak Bhawan, Bhotahity, Kathmandu (2077).

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EXAMPLE 2 (Activity No. 2)

Title:

TO VERIFY AND UNDERSTAND THE WELL-KNOWN FORMULA $\boldsymbol{\lim_{x \to 0} \frac{\tan x}{x} = 1.}$

HINT: Follow the activity and process similar as in the last example.

EXAMPLE 3 (Activity No. 3)

1. Title:

TO VERIFY THAT IF A SET HAS n ELEMENTS,
THE TOTAL NUMBER OF SUBSETS OF THE SET IS 2^n

2. Objectives:

(i) To acquaint with the basic concept of cardinality of a finite set.

(ii) To gain basic knowledge about subsets of a finite set.

3. Material Required:

Paper, different colored pencils, sharpener.

4. Theory:

Using mathematical induction, it can be proved that if a set contains exactly n elements, then it has 2^n subsets.

5. Procedure:

(i) Take a set A_0 having 0 elements and represent it as in Fig.1

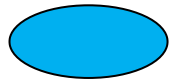

Fig.1 : Subset of $A_0 = \{\ \}$

(ii) Take a set A_1 having one element, say a_1 , and represent it as in Fig.2

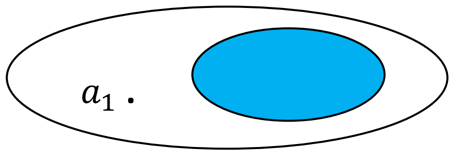

Fig.2: Subsets of $A_1 = \{ a_1 \}$

(iii) Take a set A_2 having 2 elements, say a_1 and a_2 , and represent it as in Fig.3

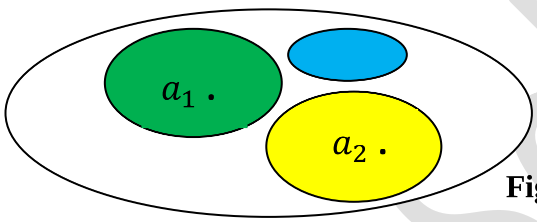

Fig.3: Subsets of $A_2 = \{ a_1 , a_2\}$

6. Observation:

In Fig.1, possible subset of A_0 is A_0 itself.
Therefore, number of subsets of set A_0 having 0 elements is $1 = 2^0$.
In Fig.2, possible subsets of A_1 are $\emptyset$ and $\{a_1\}$.
Therefore, the number of subsets of set A_1 having 1 element is $2 = 2^1$.
In Fig.3, possible subsets of set A_2 are $\emptyset$, $\{a_1\}$, $\{a_2\}$ and $\{a_1, a_2\}$.
Therefore, the number of subsets of set A_2 having 2 elements is $4 = 2^2$.
Similarly, if the set A_n contains n elements, then the total number of subsets will be 2^n.

7. Conclusion:

For a set having n elements, there are 2^n subsets of it.

8. Application:

This activity can be used for calculating the total number of subsets of a given finite set.

9. References:

(i) Bajracharya, B.C.; "Basic Mathematics" Grade XI; Sukunda Pustak Bhawan, Bhotahity, Kathmandu (2077).

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EXAMPLE 4 (Activity No. 4)

1. Title:

TO OBTAIN TRUTH VALUES OF THE DISJUNCTION " $p \vee q$ " USING SWITCH CONNECTION IN PARALLEL

2. Objectives:

(i) To develop in-depth knowledge and good theoretical background in the area of pure mathematics.

(ii) To raise interest in the field of analytical world.

3. Material Required:

Switches, electric wires, battery and bulb.

4. Theory:

If p and q represent two statements, then a compound statement of type " $p \vee q$ " (i.e. p or q) is called disjunction having truth values as shown in the following table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

5. Procedure:

(i) Connect switches S_1 and S_2 in parallel and then connect the battery and bulb to complete the circuit as shown in Fig.1:

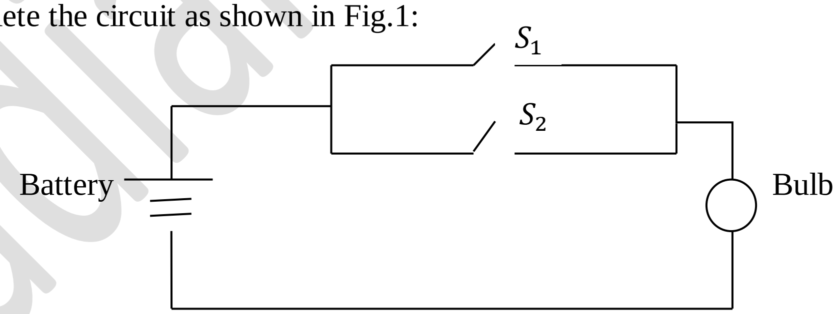

Fig.1

(ii) Let p stands for the statement " S_1 is on" and truth value of p is T.
Let q stands for the statement " S_2 is on" and truth value of q is T.
(F, if the switch is off)
The status of the bulb is represented by $p \vee q$. When it glows, the truth value is T and when it does not glow, the truth value is F.

6. Observation:

It is observed that the bulb glows if at least one of switches S_1 and S_2 is on. The detailed observation is as given below:

Switch S_1	Switch S_2	Status of bulb
On	On	Glow (Fig.2)
On	Off	Glow (Fig.3)
Off	On	Glow (Fig.4)
Off	Off	Not Glow (Fig.5)

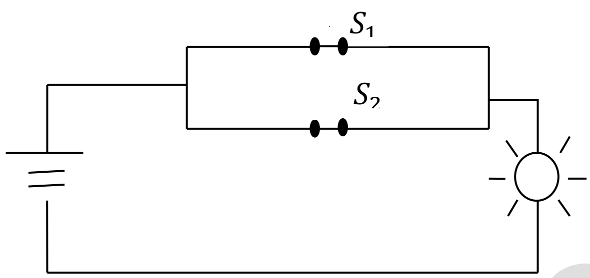

Fig.2

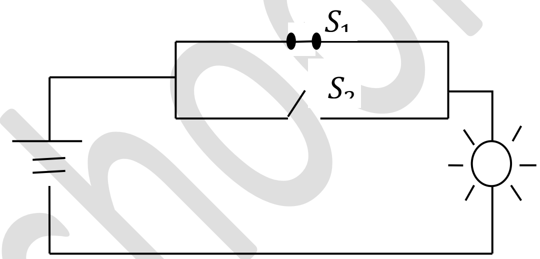

Fig.3

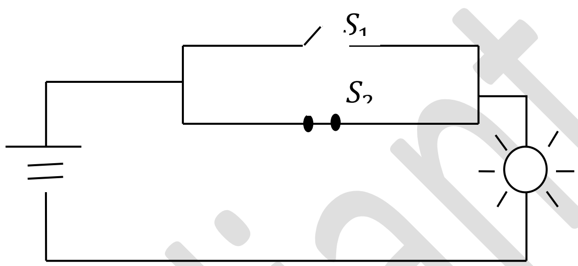

Fig.4

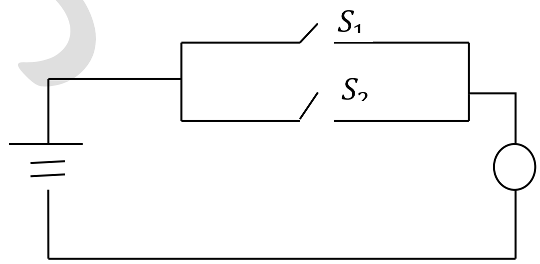

Fig.5

7. Conclusion:

The statement " $p \vee q$ " is false only when both statements p and q are false.

8. Application:

This activity helps in understanding the truth values of $p \vee q$ in different situations.

9. References

(i) Bajracharya, B.C.; "Basic Mathematics" Grade XI; Sukunda Pustak Bhawan, Bhotahity, Kathmandu (2077).

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EXAMPLE 5 (Activity No. 5)

Title:

TO OBTAIN TRUTH VALUES OF THE DISJUNCTION " $p \wedge q$ " USING SWITCH CONNECTION IN SERIES

HINT: Follow the activity and process similar as in the previous example.

EXAMPLE 6 (Activity No. 6)

1. **Title :**

TO DEMONSTRATE THAT A.M. BETWEEN TWO UNEQUAL POSITIVE NUMBERS IS ALWAYS GREATER THAN G.M.

2. **Objectives:**

(i) To understand the relation between A.M. and G.M. between two unequal positive numbers.

(ii) To explain visually the concept of A.M. and G.M.

3. **Material Required:**

Colored chart papers, sketch pens, adhesive, ruler, pencil, sharpener etc.

4. **Theory:**

The A.M. and G.M. between two unequal positive numbers a and b is given by

$$A.M. = \frac{a+b}{2} \quad \text{and} \quad G.M. = \sqrt{ab}\,.$$

And we have $A.M. > G.M.$, i.e. $\frac{a+b}{2} > \sqrt{ab}$.

5. **Procedure:**

(i) Cut four pieces of different colored chart papers of size $a \times b$, where $a = 6$ cm and $b = 4$ cm.

(ii) Arrange all these pieces as shown in Fig.1.

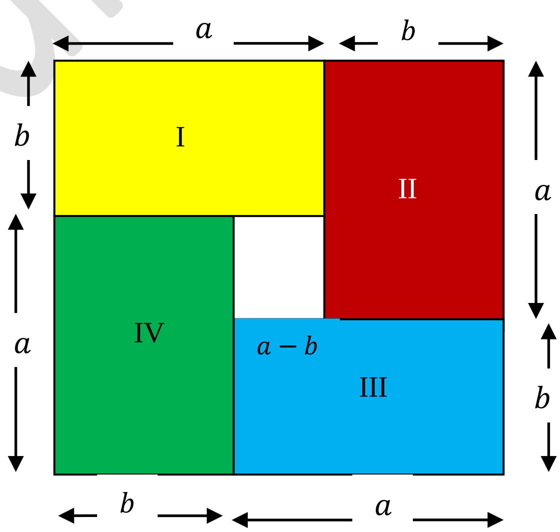

Fig.1

6. Observation:

Area of the square having side $(a+b)$

$= \text{sum of areas of rectangles I, II, III and IV} \;\; + \;\; \text{area of the square having side } (a-b)$

i.e. $(a+b)^2 = (ab+ab+ab+ab)+(a-b)^2$

i.e. $100 \text{ cm}^2 = 4 \times 6 \text{ cm} \times 4 \text{ cm} + (2 \text{ cm})^2$

Clearly, Area of the square having side$(a+b)$

$> \text{sum of areas of rectangles I, II, III and IV}$

i.e. $100 \text{ cm}^2 > 4 \times 6\, cm \times 4\, cm$

i.e. $(a+b)^2 > 4ab$

$\Rightarrow \frac{a+b}{2} > \sqrt{ab}$

$\Rightarrow A.M. > G.M.$

7. Conclusion:

For any two unequal positive numbers, $A.M. > G.M.$

8. Application:

This activity can be used in various branches of statistics, and may be helpful in proving $A.M. > G.M.$

9. References:

(i) Bajracharya, B.C.; "Basic Mathematics" Grade XI; Sukunda Pustak Bhawan, Bhotahity, Kathmandu (2077).

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EXAMPLE 7 (Activity No. 7)

1. Title:

TO FIND THE SUM OF INFINITE GEOMETRIC SERIES $1+\frac{1}{2}+\frac{1}{2^2}+\cdots$

2. Objectives:

(i) To understand the sum of the infinite geometric series.

(ii) To be able to gain basic knowledge about the sum of the infinite geometric series.

3. Material Required:

Two colored papers, scissor, adhesive, pencil, sharpener etc.

4. Theory:

The sum of the infinite geometric series $a+ar+ar^2+\cdots$ is given by

$$\frac{a}{1-r}\text{, provided } |r|<1.$$

5. Procedure:

(i) Take two pieces of different colored papers of convenient size, say 1 cm × 20 cm.

(ii) Cut a piece at the middle.

(iii) Take any one of them and again cut it at the middle.

(iv) Take any one of them and cut it at the middle and continue the process as shown in Fig.1.

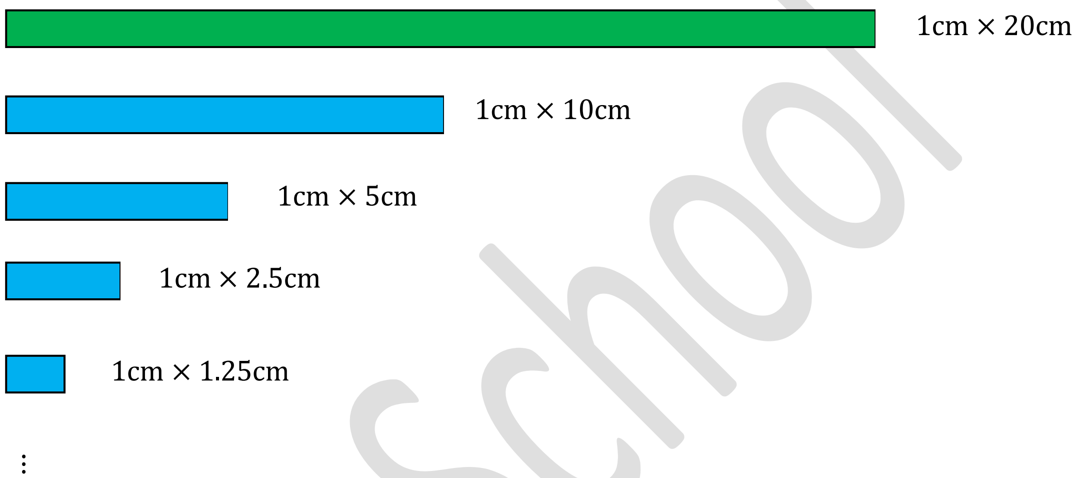

Fig.1

6. Observation:

Arrange the pieces as shown in Fig.1.

This arrangement forms a series $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots$. The sum of lengths of all pieces of paper is almost as of **2 pieces** taken at the beginning.

Also, using the formula, the sum is given by

$$\frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\left(\frac{1}{2}\right)} = 2.$$

7. Conclusion:

The sum of the infinite geometric series $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots$ is 2.

8. Application:

This activity can be used to find the sum of certain infinite geometric series.

9. References:

(i) Bajracharya, B.C.; "Basic Mathematics" Grade XI; Sukunda Pustak Bhawan, Bhotahity, Kathmandu (2077).

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EXAMPLE 8 (Activity No. 8)

1. **Title:**

   TO VERIFY THAT THE POINT OF INTERSECTION OF TWO STRAIGHT LINES $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ IS

   $$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right) \text{, PROVIDED } a_1b_2 - a_2b_1 \neq 0.$$

2. **Objectives:**

   (i) To understand the concept of point of intersection of two non-parallel straight lines.

   (ii) To explain visually, the idea of point of intersection of two non-parallel straight lines.

3. **Material Required:**

   Graph paper, pencil, sharpener, cardboard, adhesive etc.

4. **Theory:**

   The point of intersection of two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by

   $$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right), \qquad \text{provided } a_1b_2 - a_2b_1 \neq 0.$$

5. **Procedure:**

   (i) Take a cardboard of convenient size and paste a graph paper on it.

   (ii) Draw two mutually perpendicular lines to represent $x -$ axis and $y -$ axis.

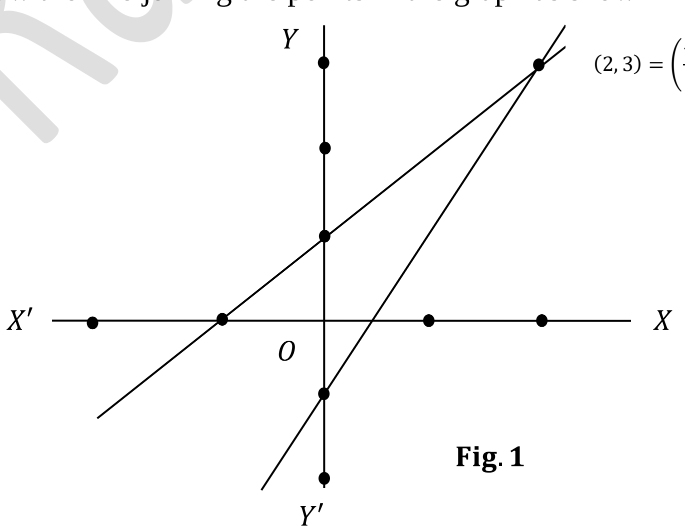
   (iii) Take a suitable scale for making points on $x -$ axis and $y -$ axis.

6. **Observation:**

   Let the equations of the two lines be $x - y + 1 = 0$ and $2x - y - 1 = 0$.

   The first line meets $x -$ axis and $y -$ axis at points (-1, 0) and (0, 1), respectively. We draw the line joining the points in the graph as shown in Fig.1.

   The second line meets $x -$ axis and $y -$ axis at points (0.5, 0) and (0, -1), respectively. We draw the line joining the points in the graph as shown in Fig.1.



$$(2,3) = \left(\frac{-1\cdot(-1) - (-1)\cdot 1}{1\cdot(-1) - 2\cdot(-1)}, \frac{1\cdot 2 - (-1)\cdot 1}{1\cdot(-1) - 2\cdot(-1)}\right)$$

Fig. 1

From the graph, it can be observed that the point of intersection is $(2, 3)$.
Also, using the formula,

$$\left(\frac{-1 \cdot (-1) - (-1) \cdot 1}{1 \cdot (-1) - 2 \cdot (-1)}, \frac{1 \cdot 2 - (-1) \cdot 1}{1 \cdot (-1) - 2 \cdot (-1)}\right) = (2, 3).$$

7. Conclusion:

The point of intersection of two intersecting may be obtained by plotting the lines in the graph paper.

8. Application:

This activity can be used to find the point of intersection of two intersecting lines.

9. References:

(i) Bajracharya, B.C.; "Basic Mathematics" Grade XI; Sukunda Pustak Bhawan, Bhotahity, Kathmandu (2077).

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